4. V. A. Ditkin and A. P. Prudnikov, *Spracovnik po operacionnomu izcisleniu* (in Russian), p. 265. Vysch. skola, Moskva (1965).
5. M. M. Klimenko, R. E. Krizanovski and V. E. Sherman, Impulznyj metod opredelenia Temperaturoprovodnosti, (in Russian), *Teplofiz. Vysok Temp.* **17,** 1216 (1979).
6. A. V. Lykov, *Teplomassoobmen* (in Russian), p. 451. Energia, Moskva (1978).
7. R. J. Gutfeld and A. H. Nethercot, *Phys. Rev. Lett.* **12,** 641 (1964); R. J. Gutfeld, *Physical Acoustics*, Vol. 5, p. 223. Academic Press, New York (1968).

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Integral analysis of conjugate natural convection heat transfer from a long, vertical fin

K. Himasekhar
Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, 111A Towne Building/D3, Philadelphia, PA 19104, U.S.A.

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INTRODUCTION

Buoyancy-induced convection is of great importance in many heat removal processes in technology. In particular, for low-power-level devices, it may be a significant cooling mechanism. In such cases, the transfer surface area may be increased, as in fins, for augmentation of heat transfer rates. In the thermal analysis of vertical fins, it is usually assumed that the fin is isothermal. This may be a reasonable assumption for short fins with high thermal conductance. However, long fins with low conductance would not be isothermal and for the estimation of heat transfer rates from such fins, the conjugate problem of conduction within the fin has to be solved simultaneously with natural convection in the ambient fluid. A numerical solution of this problem for a short plate fin in a fluid with $Pr = 0.72$ was obtained by Sparrow and Acharya [1]. Lock and Gunn [2] developed a similarity solution for a short, tapered fin in a fluid of infinite Prandtl number. Recently, Kuehn *et al.* [3] presented a similarity solution for the conjugate free convection heat transfer from a vertical fin of infinite length and obtained results for a uniform conductivity plate fin as a function of the fluid Prandtl number.

In the present work, an integral analysis has been carried out to obtain a closed-form solution for the heat transfer rates from a long, vertical fin with variable conductivity and/or thickness. The solution for the special case of fin with constant thickness and conductivity has been compared with that of Kuehn *et al.* [3] and a close agreement confirms the utility of the proposed equation.

ANALYSIS AND RESULTS

Consider an infinitely long, vertical fin as shown in Fig. 1. The coordinate system used is also depicted in this figure. The base of the fin can be selected arbitrarily if the corresponding temperature T_b is known [3]. The fin is at a higher temperature than that of the ambient fluid. The flow is assumed to be laminar. The Boussinesq approximation for the density variation is employed and the other fluid properties are taken to be constant. Further details of the problem can be found in ref. [3].

In this paper the case of the fin being hotter than the ambient fluid is explicitly considered. However, the analysis as well as the results also apply to the case of the fin being colder than the bulk fluid. In the latter case, the fin is to be inverted.

The conservation equations for the fluid in the integral

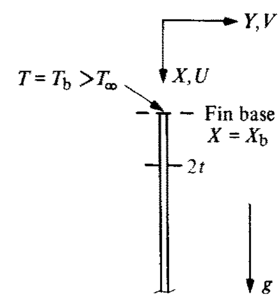

Fig. 1. The physical model.

form are as follows [4]:

$$\frac{d}{dX}\int_0^\delta U^2\,dY = -\nu\left.\frac{\partial U}{\partial Y}\right|_{Y=0} - \int_0^\delta g\beta(T-T_\infty)\,dY \quad (1)$$

and

$$\frac{d}{dX}\int_0^\delta U(T-T_\infty)\,dY = -\alpha\left.\frac{\partial T}{\partial Y}\right|_{Y=0}. \quad (2)$$

Energy balance for the fin with varying thickness and/or conductivity yields:

$$\frac{d}{dX}\left(K_F t\frac{dT_F}{dX}\right) - h(T_F - T_\infty) = 0. \quad (3)$$

Here the thin fin approximation is employed. The boundary conditions for the fin are:

$$T_F = T_b \quad \text{at } X = X_b \quad (4a)$$

$$T_F = T_\infty \quad \text{as } X \to \infty. \quad (4b)$$

The temperature variation of the fin is taken as the power-law of the type [3]:

$$(T_F - T_\infty) = (T_b - T_\infty)(X/X_b)^n. \quad (5)$$

The following velocity and temperature distributions are assumed for the fluid [4]:

$$U/U_r = (Y/\delta)(1-Y/\delta)^2 \quad (6)$$

$$(T-T_\infty) = (T_F - T_\infty)(1-Y/\delta)^2. \quad (7)$$

Equations (1)–(3) are solved by using the following power-law variations for the boundary-layer thickness (δ) and reference velocity (U_r):

$$\delta = c_1 x^{P1} \quad (8)$$

$$U_r = -c_2 x^{P2}. \quad (9)$$

NOMENCLATURE

h	local heat transfer coefficient
K	fluid thermal conductivity
K_F	fin thermal conductivity
L	length of the fin along the base
Nu	local Nusselt number, hX_b/K
Pr	Prandtl number, ν/α
q_t	dimensionless total heat transfer per unit length along the base, $Q_t/[LK(T_b-T_\infty)]$
Ra	Rayleigh number, $g\beta X_b^3(T_b-T_\infty)/(\nu\alpha)$
Ra^*	modified Rayleigh number, $Ra(t_b/X_b)^3$
t	fin half thickness
U	vertical velocity
u	dimensionless vertical velocity, UX_b/α
V	horizontal velocity
v	dimensionless horizontal velocity, VX_b/α
X	vertical coordinate
X_b	location of fin base given by equation (12)
x	dimensionless vertical coordinate, X/X_b
Y	horizontal coordinate
y	dimensionless horizontal coordinate, Y/X_b.

Greek symbols

β	coefficient of thermal expansion
δ	dimensionless boundary-layer thickness
θ	dimensionless temperature, $(T-T_\infty)/(T_b-T_\infty)$.

Subscripts

b	fin base
F	fin
r	reference quantity
t	total quantity
∞	ambient fluid.

The solution to equations (1) and (2) would then yield

$$\left.\begin{aligned} P1 &= (1-n)/4 \\ P2 &= (1+n)/2 \\ Z &= 80/(5n+3) \\ C_2 &= \sqrt{\left(\frac{-Z\,Ra\,Pr}{Pr+(Z/35)(3n+5)/4}\right)} \\ C_1 &= (-3Z/C_2)^{1/2}. \end{aligned}\right\} \quad (10)$$

The solution to equation (3) for conductivity–thickness product of the fin of the form:

$$K_F t = K_{F,b} t_b \cdot x^m \quad (11)$$

as indicated in [3], is manipulated to obtain the following:

$$n = 4m-7$$

$$X_b/t_b = (Ra^*)^{1/7}[4(K_{F,b}/K) \times (4m-7)(5m-8)/-\theta'(0)]^{4/7} \quad (12)$$

where $Ra^* = g\beta(T_b-T_\infty)t_b^3/(\nu\alpha)$.

From equations (10) and (12), the following results in non-dimensional form are derived:

Temperature gradient:

$$-\theta'(0) = \{16(8-15m)/45[1+(16-12m)/7(8-5m)Pr]\}^{1/4}. \quad (13)$$

Fin local Nusselt number:

$$Nu = (Ra^*)^{1/4}[-\theta'(0)]x^{(m-2)}(x_b/t_b)^{3/4}/2^{1/2}. \quad (14)$$

Fin total heat transfer:

$$q_t = (K_{F,b}/K)^{3/7}(14-8m)[(7-4m)(8-5m)]^{-4/7} \times \left\{\frac{4(8-15m)Ra^*}{45[1+(16-12m)/7Pr(8-5m)]}\right\}^{1/7}. \quad (15)$$

It is to be noted that only for m less than 8/15, do the above equations give physically meaningful results. The value of $m = 0$ corresponds to a constant-property plate fin considered in ref. [3]. From equation (15), the total heat transferred from the fin to the ambient fluid, for this special case, would be

$$(q_t)_{m=0} = 1.34(K_{F,b}/K)^{3/7}[Ra^*/(1+2/7Pr)]^{1/7}. \quad (16)$$

Even though the limit of applicability of the integral method used in the present analysis is that Pr should be around unity, the values of q_t predicted by the above equation agree to within 9% with those obtained by Kuehn *et al.* [3] for $0.1 \leqslant Pr \leqslant 100$.

A comparison of the assumed temperature and velocity profiles is also made with the similarity solutions obtained by Kuehn *et al.* [3]. The plots are not included here to conserve space. The agreement is found to be fairly good. The general trend of agreement between these profiles and the numerical solutions is observed to be essentially similar to that of Squire [5].

To demonstrate more clearly the effects of the governing parameters on the fin total heat transfer (q_t), the result in equation (15) is plotted in Fig. 2. The variation of q_t with $-m$ for different values of Prandtl numbers is shown in this figure. It is to be noted that the case of negative values of m may correspond to a fin with decreasing thickness and/or to a fin material whose conductivity increases as a power-law with temperature. As expected, for these cases, the total heat transfer is observed to increase with $-m$ increasing.

CONCLUDING REMARKS

Thus the present analysis gives an approximate analytical solution to the problem of conjugate natural convection heat transfer from a long, vertical fin with a variable conductivity and/or thickness. The heat transfer dependence on the governing parameters of the problem is well defined. Even though the results are obtained under the assumption that the fin is infinitely long, they may also be employed for

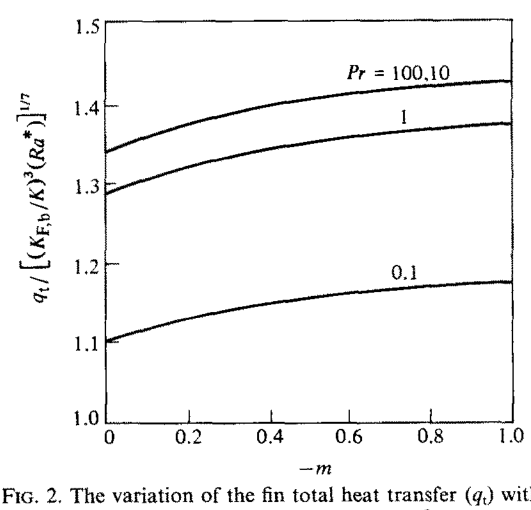

FIG. 2. The variation of the fin total heat transfer (q_t) with $-m$ for various values of Prandtl numbers.

the finite length fins under the conditions that the fin tip temperature is nearly equal to the ambient fluid temperature and the heat transfer near the tip is negligible compared to the total fin heat transfer.

REFERENCES

1. E. M. Sparrow and S. Acharya, A natural convection fin with a solution-determined nonmonotonically varying heat transfer coefficient, *Am. Soc. mech. Engrs, Series C, J. Heat Transfer* **103**, 218–225 (1981).
2. G. S. H. Lock and J. C. Gunn, Laminar free convection from a downward-projecting fin, *Am. Soc. mech. Engrs, Series C, J. Heat Transfer* **90**, 63–70 (1968).
3. T. H. Kuehn, S. S. Kwon and A. K. Tolpadi, Similarity solution for conjugate natural convection heat transfer from a long vertical plate fin, *Int. J. Heat Mass Transfer* **26**, 1718–1721 (1983).
4. Y. Jaluria, *Natural Convection Heat and Mass Transfer*, p. 73. Pergamon Press, New York (1980).
5. H. B. Squire, *Modern Developments in Fluid Dynamics* (Edited by S. Goldstein), Vol. 2, pp. (14)641–(14)643. Dover, New York (1965).

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On the heat transfer characteristics of constrained air jets impinging on a flat surface

M. SHOUKRI and A. CALKA
Department of Mechanical Engineering, McMaster University, Hamilton, Ontario, Canada L8S 4L7

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1. INTRODUCTION

IMPINGING jets is a well-known and widely used technique for realizing high heat transfer rates between a fluid and a surface. The low cost and the fine degree of control that gas jets permit have made them particularly attractive for cooling applications. Specific applications include the cooling of the leading edge of turbine blades, the cooling of electrical equipment, the annealing of metal and plastic sheets, the tempering of glass, and the drying of textiles, veneer, paper and film materials. Different jet-impingement surface configurations varying from a single circular jet to arrays of round or slot nozzles are described in ref. [1] which is the most comprehensive survey of jet-impingement heat transfer available at the present time. Reference [1], however, does not contain any mention of constrained jets, which are the focus of the present work. By constrained jet, it is meant that the flow is forced to flow back after impinging on the surface rather than spreading and flowing over the surface. It is used in applications where only localized cooling is desired as shown in Fig. 1.

In order to efficiently design jet-cooling systems, one needs to know the dependence of heat transfer rates on variables which fully characterize the system. For single round and slot nozzles, this dependence can be described in the following dimensionless form [1]:

$$\overline{Nu} = f(Re, Pr, r/D, H/D). \tag{1}$$

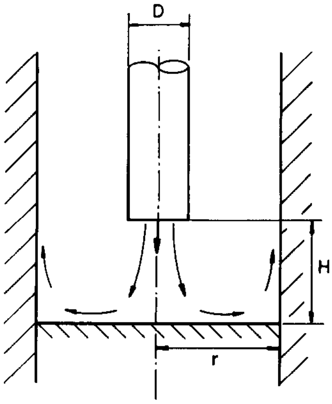

FIG. 1. A constrained, impinging jet.

Schlünder and Gnielinski [2] correlated their measurements and those of other researchers for free impinging single round jets by the following empirical equation:

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{D}{r}\,\frac{1-1.1D/r}{1+0.1(H/D-6)D/r}\,F(Re). \tag{2}$$

The function $F(Re)$ may be represented by the following expression:

$$F(Re) = 2\,Re^{1/2}\left(1+\frac{Re^{0.55}}{200}\right)^{0.5}. \tag{3}$$

Equations (2) and (3) are valid in the ranges

$$2000 \leqslant Re \leqslant 400{,}000$$
$$2.5 \leqslant r/D \leqslant 7.5$$
$$2 \leqslant H/D \leqslant 12.$$

Equation (2), in general, would not give satisfactory results for constrained jets because of the difference in flow characteristics between free and constrained impinging jets. At present, there are no references in the literature concerning the case of impinging gas jet heat transfer with a solid boundary to constrain the radial flow of gas. The present work is aimed at examining the influence of Re, H/D and r/D on the heat transfer coefficient in the case of a constrained circular air jet impinging on a surface.

2. EXPERIMENTAL ARRANGEMENT AND TEST CONDITIONS

The experimental set-up, which allowed the determination of the average heat transfer coefficient between constrained air jets and a flat heated surface is schematically shown in Fig. 2. The heat transfer surface was that of a heated cylindrical copper block. The copper block was heated using a cartridge heater and was heavily insulated to ensure one-dimensional heat flow normal to the heat transfer surface. The copper block was instrumented with four copper–constantan thermocouples located on the centerline 3, 16, 28.5 and 41 mm away from the surface. The surface temperature was obtained by extrapolating the measured linear axial temperature distribution. The heat flux was obtained by measuring the input power to the heater or by using the known thermal conductivity of the copper specimen and the measured axial temperature gradient. The two methods yielded essentially the same result (within 5%).